Introduction to Programming and Data Structures

Sorting, Searching

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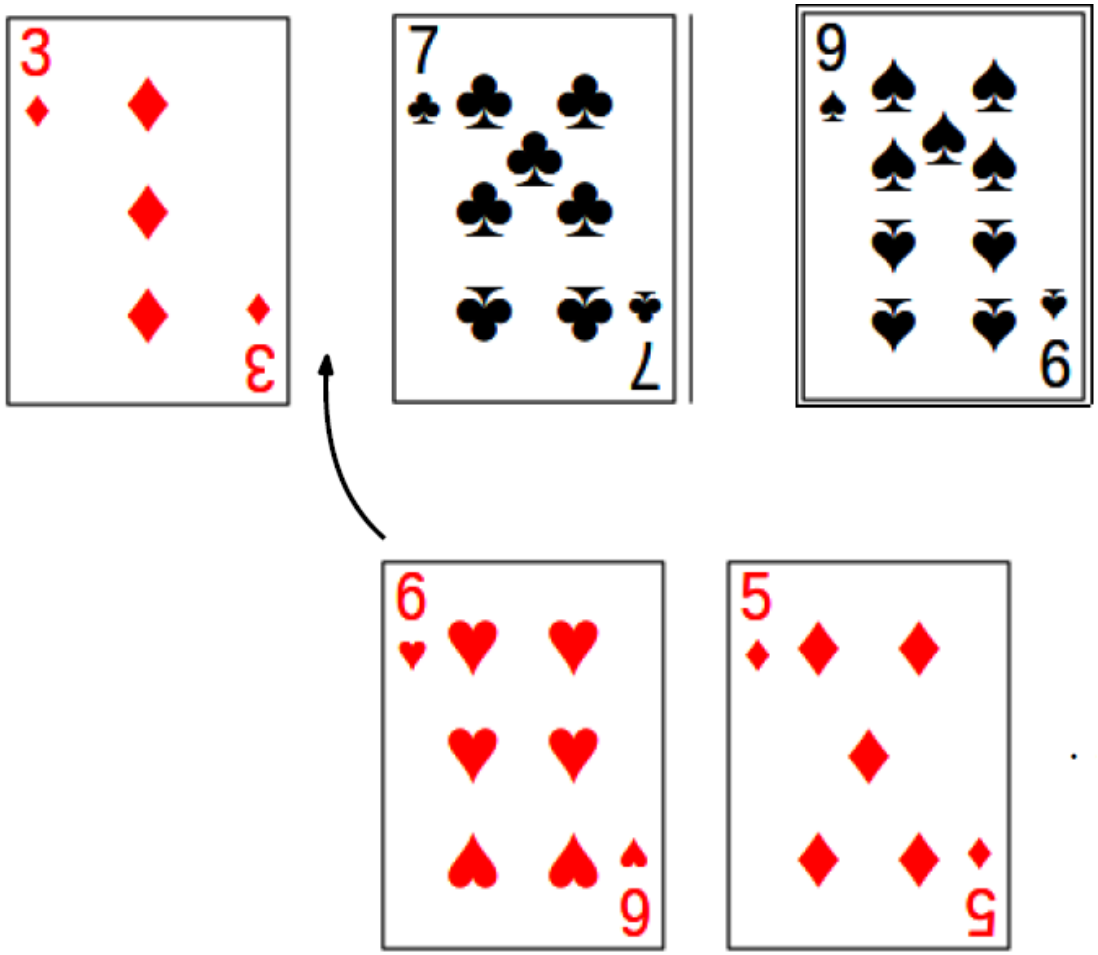
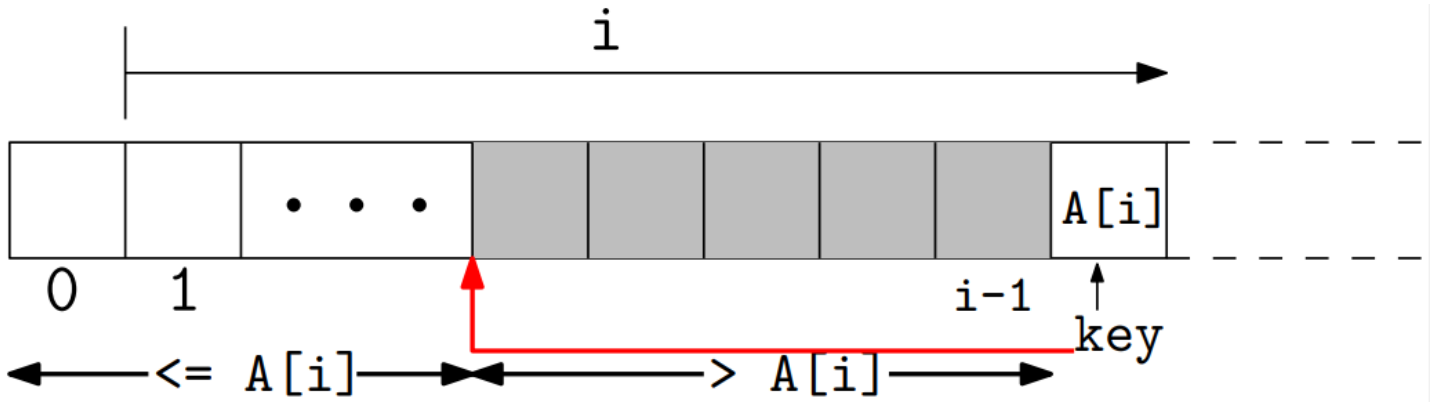
Bubble sort

```
LIST = [2, 6, 4, 8, 1]
n = len(LIST)
for i in range(n-1):
    for j in range(n-i-1):
        if LIST[j+1] < LIST[j]:
            LIST[j], LIST[j+1] = LIST[j+1], LIST[j]
```

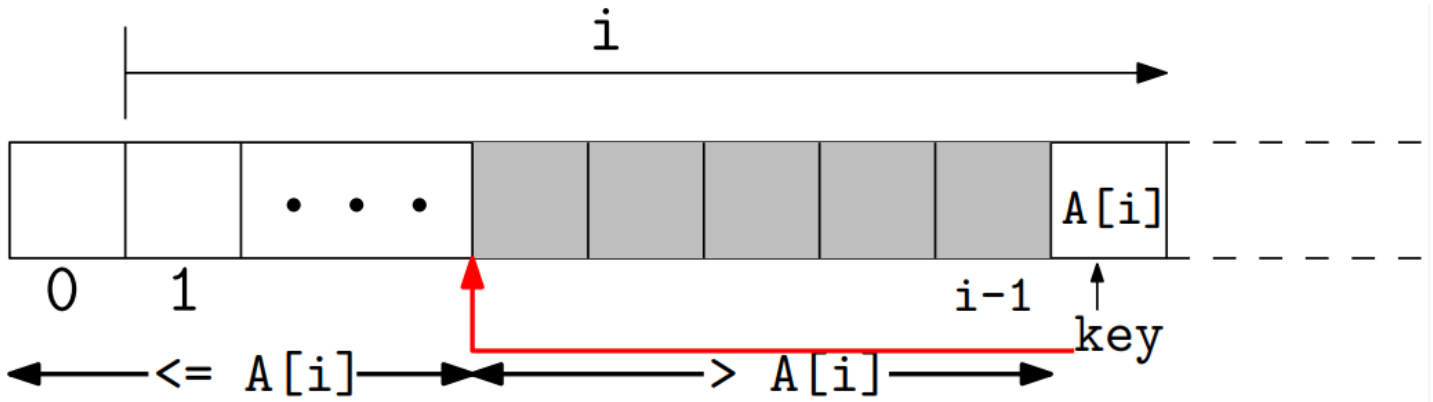
Insertion sort



Insertion sort

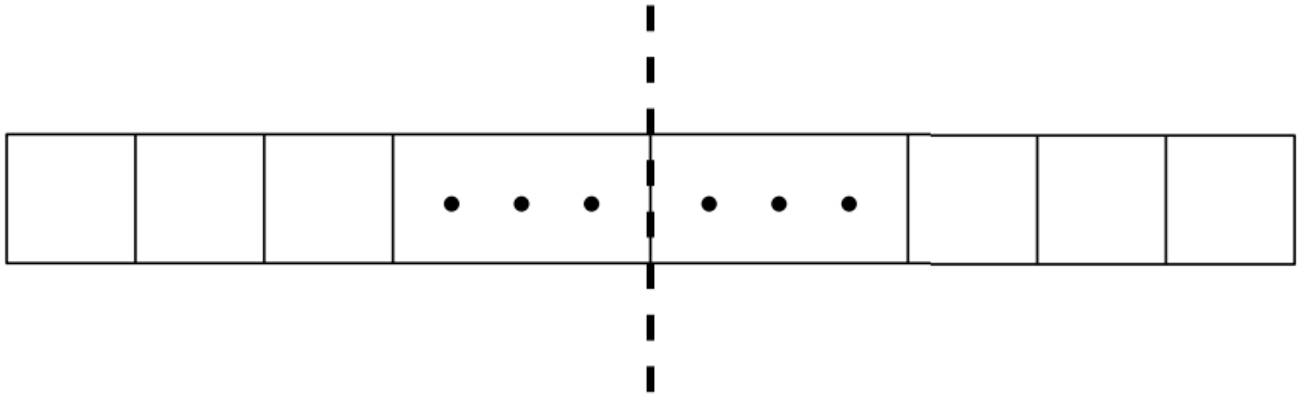


Insertion sort

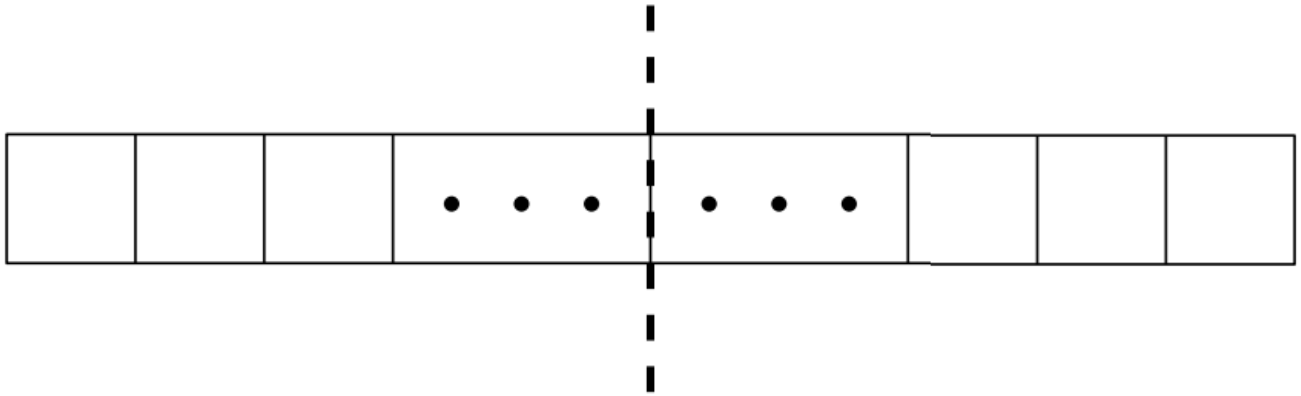



```
LIST = [2, 6, 4, 8, 1]
n = len(LIST)
for i in range(1, n):
    key = LIST[i]
    # Find the right place for LIST[i] in LIST[0 ... i-1]
    j = i - 1
    while j >= 0 and LIST[j] > key:
        LIST[j+1] = LIST[j]
        j = j - 1
    LIST[j+1] = key
```

Merge sort



Merge sort

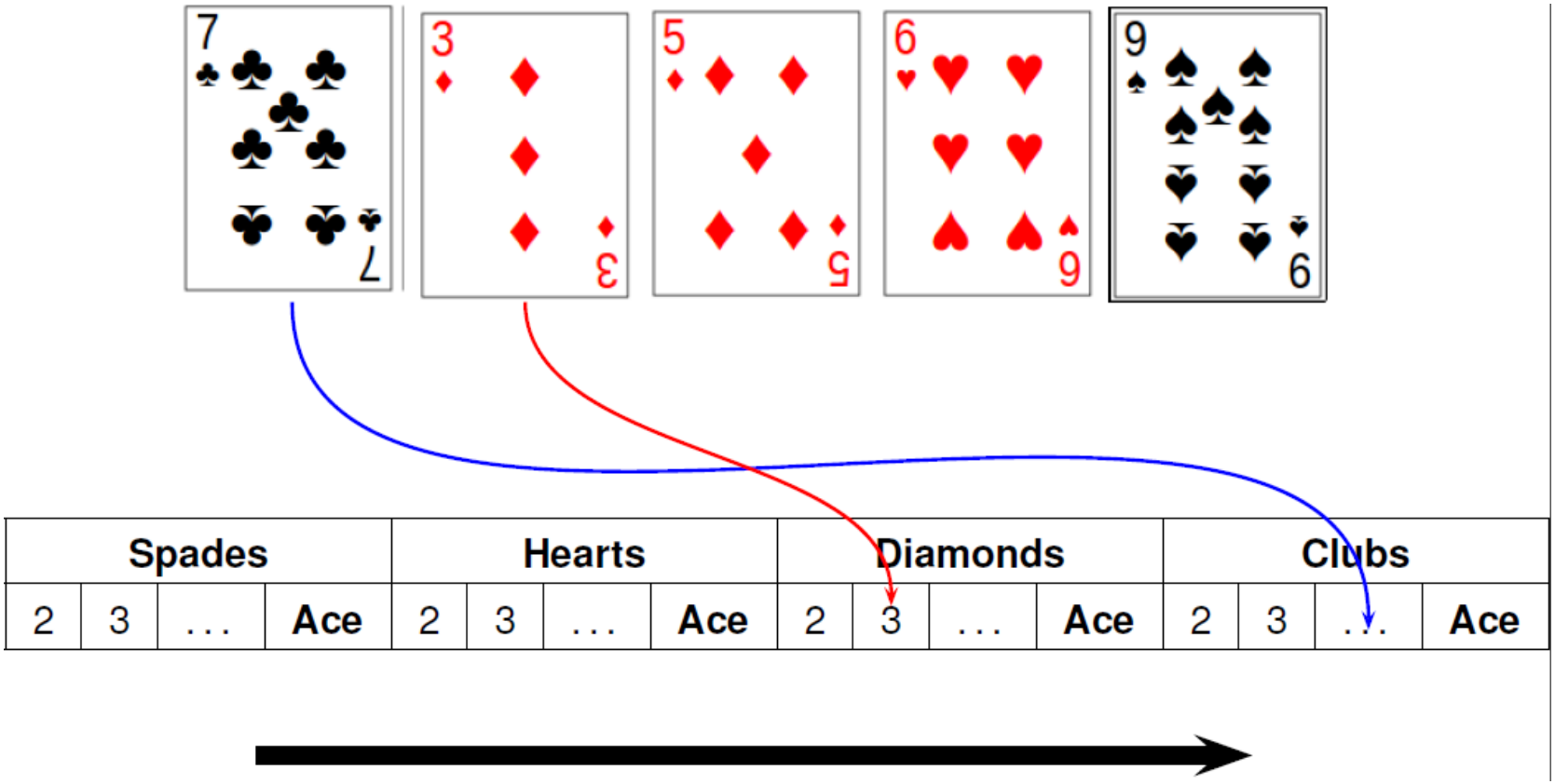


```
def msort(LIST, beginning, end):
    if beginning < end:
        middle = (beginning + end) / 2
        msort(LIST, beginning, middle)
        msort(LIST, middle+1, end)
        merge(LIST, beginning, middle, end)
```

Merge sort

```
def merge(LIST, b, m, e):
    i, j, k = b, m + 1, 0
    while i <= m and j <= e:
        if LIST[i] < LIST[j]:
            AUXLIST[k] = LIST[i]
            k, i = k+1, i+1
        else if A[i] > A[j]:
            AUXLIST[k] = LIST[j]
            k, j = k+1, j+1
        else:
            AUXLIST[k] = LIST[i]
            k, i = k+1, i+1
            AUXLIST[k] = LIST[j]
            k, j = k+1, j+1
     while i <= m:
        AUXLIST[k] = LIST[i]
        k, i = k+1, i+1
     while j <= e:
        AUXLIST[k] = LIST[j]
```

Bucket sort


7
3
5
6
9
Spades
Hearts
Diamonds
Clubs
2 3 . . . Ace
2 3 . . . Ace
2 3 . . . Ace
2 3 . . . Ace

Bucket sort

```
# Allocate space for array A
for i in range(n):
    A[i] = i
```

Timsort

Timsort is a highly optimized and stable version of merge sort that effectively combines insertion sort.

Timsort was implemented by Tim Peters in 2002 for use in the Python programming language.

- `https://en.wikipedia.org/wiki/Timsort`
- `https://realpython.com/sorting-algorithms-python/#pythons-built-in-sorting-algorithm`

Timsort

Input: Unsorted collection
Output: Sorted collection
`// Calculate run length`
$runLength$:= calculateRunLength($array$);
`// Perform insertion sort on each run`
for $start \leftarrow 0$ **to** $array.length$ **by** $runLength$ **do**
    $end := \min(array.length - 1, start + runLength - 1)$;
    insertionSort($array$, $start$, end);
end
`// Recursively merge adjacent runs`
$mergeSize := runLength$;
while $mergeSize < array.length$ **do**
    for $left \leftarrow 0$ **to** $array.length$ **by** $size * 2$ **do**
        $mid := left + size - 1$;
        $right := \min(array.length - 1, left + (2 * size) - 1)$;
        if $mid < right$ **then**
            merge($array$, $left$, mid, $right$);
        end
    end
    $mergeSize := mergeSize * 2$;
end

The searching problem

Given a data structure S over a domain of data items D, verify (search) whether a given data item s belongs to D or not. If it belongs, then return the position (index) of s in D with respect to its organization in S.

Linear search

Let $LIST[] = \{30, 10, 20, 40, 50\}$ and $s = 40$.

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$i = 0$	↓					
Whether $LIST[i] = s$	30	10	20	40	50	FALSE

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Whether $LIST[i] = s$	30	10	20	40	50	FALSE
$i = i + 1$		↓				
Whether $LIST[i] = s$	30	10	20	40	50	FALSE

Linear search

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Whether $LIST[i] = s$	30	10	20	40	50	FALSE
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$i = i + 1$			↓			
Whether $LIST[i] = s$	30	10	20	40	50	FALSE
$i = i + 1$				↓		
Whether $LIST[i] = s$	30	10	20	40	50	TRUE

Return $i = 3$.

Jump search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Further assume that the block size (of jump) is $b = 2$.

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$pre\_i = 0, i = b$			↓			
Whether $LIST[i] < s$	10	20	30	40	50	TRUE

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$pre\_i = 0, i = b$			↓			
Whether $LIST[i] < s$	10	20	30	40	50	TRUE
$pre\_i = i, i = i + b$					↓	
Whether $LIST[i] < s$	10	20	30	40	50	FALSE

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$pre\_i = i, i = i + b$					↓	
Whether $LIST[i] < s$	10	20	30	40	50	FALSE
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$pre\_i = i, i = i + b$					↓	
Whether $LIST[i] < s$	10	20	30	40	50	FALSE
$pre\_i = pre\_i + 1$				↓		
Whether $LIST[pre\_i] = s$	10	20	30	40	50	TRUE

Note: The block size in jump search is taken as $b = \sqrt{n}$, where n denotes the size of the list.

Binary search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

$l = 0, r = 4, i = l + (r - l)/2 = 2$			↓			
Whether $LIST[i] = s$	10	20	30	40	50	FALSE

Binary search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

$l = 0, r = 4, i = l + (r - l)/2 = 2$			↓			
Whether $LIST[i] = s$	10	20	30	40	50	FALSE
			↓			
Whether $LIST[i] < s$	10	20	30	40	50	TRUE

Binary search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

$l = 0, r = 4, i = l + (r - l)/2 = 2$			↓			
Whether $LIST[i] = s$	10	20	30	40	50	FALSE
			↓			
Whether $LIST[i] < s$	10	20	30	40	50	TRUE
$l = i+1, i = l+(r-l)/2 = 3$				↓		
Whether $LIST[i] = s$	10	20	30	40	50	TRUE

Comparing linear/jump/binary search

	Linear search	**Jump search**	**Binary search**
Requirements	Nothing	Ordered list	Ordered list
Backward scan required	No	Once	Multiple times
Time complexity (best case)	$O(1)$	$O(b)$	$O(1)$
Time complexity (average case)	$O(n)$	$O(\sqrt{n})$	$O(\log n)$
Time complexity (worst case)	$O(n)$	$O(\sqrt{n})$	$O(\log n)$

Interpolation search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

$i = 0 + (40 - 10) * (4 - 0)/(50 - 10) = 3$				↓		
Whether $LIST[i] = s$	10	20	30	40	50	TRUE

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$.

$i = 0 + (40 - 10) * (4 - 0)/(50 - 10) = 3$				↓		
Whether $LIST[i] = s$	10	20	30	40	50	TRUE

Note: The interpolation search is efficient over binary search for the ordered lists having uniformly distributed data items.

Fibonacci search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Note that, the index of largest Fibonacci number that is greater than or equal to the length of given array is $f = 5$.

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Note that, the index of largest Fibonacci number that is greater than or equal to the length of given array is $f = 5$.

$i = Fibonacci(f-2) = 2$			↓			
Whether $LIST[i] = s$	10	20	30	40	50	FALSE

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Note that, the index of largest Fibonacci number that is greater than or equal to the length of given array is $f = 5$.

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Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Note that, the index of largest Fibonacci number that is greater than or equal to the length of given array is $f = 5$.

$i = Fibonacci(f-2) = 2$			↓			
Whether $LIST[i] = s$	10	20	30	40	50	FALSE
			↓			
Whether $LIST[i] < s$	10	20	30	40	50	TRUE
$f = 4-2+1 = 3, i = i + Fibonacci(f-2) = 3$				↓		
Whether $LIST[i] = s$	10	20	30	40	50	TRUE

Fibonacci search

Let $LIST[] = \{10, 20, 30, 40, 50\}$ and $s = 40$. Note that, the index of largest Fibonacci number that is greater than or equal to the length of given array is $f = 5$.

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$f = 4-2+1 = 3, i = i + Fibonacci(f-2) = 3$				↓		
Whether $LIST[i] = s$	10	20	30	40	50	TRUE

Note: Fibonacci search is a suitable replacement of binary search where the list is unbounded. Unlike binary search, it does not use the costly division operation (though avoidable with bitwise shift).

Problems

1. Let us define a sequence $a_0, a_1, a_2, \ldots, a_{n-1}$ as Λ-*bitonic* if there exists a j, $0 \leq j < n$, such that $a_0 < a_1 < \cdots < a_j > a_{j+1} > \cdots > a_{n-1}$. Consider an Λ-*bitonic* array consisting of integer entries. Write a program to efficiently find the largest element of the array.